

Domain wall switching: optimizing the energy landscape

Zhihong Lu

MINT Center and Department of Physics and Astronomy
University of Alabama, Tuscaloosa AL 35487-0209
Email: zlu@mint.ua.edu

P. B. Visscher

MINT Center and Department of Physics and Astronomy
University of Alabama, Tuscaloosa AL 35487-0209
Email: visscher@ua.edu

W. H. Butler

MINT Center and Department of Physics and Astronomy
University of Alabama, Tuscaloosa AL 35487-0209
Email: wbutler@mint.ua.edu

Abstract—It has recently been suggested that exchange spring media offer a way to increase media density without causing thermal instability (superparamagnetism), by using a hard and a soft layer coupled by exchange. Victora has suggested a figure of merit $\xi = 2E_b/\mu_0 m_s H_{sw}$, the ratio of the energy barrier to that of a Stoner-Wohlfarth system with the same switching field, which is 1 for a Stoner-Wohlfarth (coherently switching) particle and 2 for an optimal two-layer composite medium. A number of theoretical approaches have been used for this problem (e.g., various numbers of coupled Stoner-Wohlfarth layers and continuum micromagnetics). In this paper we show that many of these approaches can be regarded as special cases or approximations to a variational formulation of the problem, in which the energy is minimized for fixed magnetization. The results can be easily visualized in terms of a plot of the energy E as a function of magnetic moment m_z , in which both the switching field [the maximum slope of $E(m_z)$] and the stability (determined by the energy barrier ΔE) are geometrically visible. In this formulation we can prove a rigorous limit on the figure of merit ξ , which can be no higher than 4. We also show that a quadratic anisotropy suggested by Suess *et al* comes very close to this limit.

I. INTRODUCTION

Recently the concept of an exchange-spring medium[1], [2], [3] whose grains have a soft and a hard layer has been generalized[4] to a system with a continuously-varying anisotropy. Various models of this system have been explored – the purpose of this paper is to show that the relationships between these can be easily visualized by using a variational formulation of the problem.

In an exchange-spring medium, we want to minimize the switching field, but of course we can make this as small as we want by using a very small anisotropy, and the medium will be superparamagnetic (thermally unstable) and useless. To make comparisons between media, we must hold something constant to maintain stability. Often what is held constant is the anisotropy field (the coercivity $2K/M_s$) of the hardest layer. However, this hardest layer might be very thin and have little

effect on the overall coercivity. In a practical application the quantity it is most important to hold constant is the overall energy barrier to switching, which we will assume determines the thermal stability of the medium. Victora[1] introduced a figure of merit for this purpose, $\xi = 2E_b/\mu_0 m_s H_{sw}$ (here E_b and m_s are the barrier energy and saturation magnetic moment per unit area) for which we will prove a rigorous bound in Sec. III.

To do this, we develop a variational formulation in which we describe the switching behavior in terms of a function $E(m_z)$, the energy per unit area as a function of the magnetic moment per unit area. This turns out to be a very useful way of thinking about switching problems.

II. MODEL

We consider a one-dimensional model, in which the magnetization $\mathbf{M}(z)$ is a function only of one variable z (independent of x and y). We will allow the anisotropy $K(z)$, exchange constant $A(z)$, and saturation magnetization $M_s(z)$ to vary arbitrarily with z . Since we will do computations with a discrete approximation to this continuum model (which approaches the continuum model as the cell size $\rightarrow 0$), we will write the energy in a discrete form. It has cells labeled by i , with magnetization vectors \mathbf{M}_i . In the quasistatic energy minima we will consider, these vectors will lie in a plane, so they can be described by giving the angle θ_i of the magnetization relative to the long axis of the grain (the z axis):

The energy (per unit area in the xy plane) E of our system is then given in terms of the values of K and M at each cell (and A between each neighboring pair of cells) by

$$E = \sum_{i=1}^N a_i K_i \sin^2 \theta_i + \sum_{i=1}^{N-1} a_i \frac{2A_{i,i+1}}{a_{i,i+1}^2} \cos(\theta_{i+1} - \cos \theta_i) + \sum_{i=1}^N a_i \mu_0 M_i H \cos \theta_i \quad (1)$$

where K_i is the (perpendicular) anisotropy at the center of cell i , a_i is the length of cell i , $a_{i,i+1}$ is the distance between cells i and $i+1$, $A_{i,i+1}$ is the continuum exchange parameter evaluated between these cells[5], and H is the external field (assumed along z). For simplicity, we do not consider magnetostatic energy here – in similar systems, micromagnetic simulation has shown that this affects the coercivity by only a few percent.

We consider here quasistatic switching - we assume that we vary H in such a way that the system is always at a relative minimum (with respect to the \mathbf{M}_i s) of the energy. More precisely, we assume H is very slightly above this value, so that the magnetic moment

$$m_z = \sum_i^N a_i M_i \cos \theta_i \quad (2)$$

increases slowly, and we consider the limit in which the rate of increase approaches zero. Note that this is never true in a real switching event - after a domain wall has traversed most of the sample, it would require reversing H to keep the system quasistatic. However, by this time it is irrelevant whether the system remains quasistatic (it will finish switching in either case) and in the initial stages the quasistatic assumption is often reasonable.

It would appear that to find the quasistatic switching trajectory, in which H varies with time, we would need to minimize a function $E(\theta_1, \theta_2, \dots, \theta_N, H)$ of a large number of variables $\theta_1, \theta_2, \dots, \theta_N$, for each value of H independently. However, there is a way around this. We can choose some coordinate in the space of θ_i s (we choose the longitudinal component of the magnetic moment, m_z , for reasons apparent below) and first minimize $E(\theta_1, \theta_2, \dots, \theta_N, H)$ for fixed m_z , obtaining a function (the constrained minimum energy) $E(m_z, H)$. Then we can minimize $E(m_z, H)$ with respect to m_z , obtaining the same relative minimum $E(H)$ we would have obtained by unconstrained minimization. [Note that there may be more than one relative minimum, so we should call this $E_j(H)$ where j indexes the minima, but we will omit this index for simplicity.] The advantage of this apparently-circuitous method of finding the minimum is that the configuration minimizing $E(\theta_1, \theta_2, \dots, \theta_N, H)$ is actually independent of H ! This is apparent from Eq. (1) above, since the only dependence on H is the Zeeman term $\mu_0 m_z H$, which is a constant when m_z is held fixed. The result is that we need only compute the constrained minimum energy at $H = 0$, and it is given at any other field H by

$$E(m_z, H) = E(m_z, 0) - \mu_0 m_z H \quad (3)$$

Furthermore, this energy is minimized at a particular H by setting $\partial E(m_z, H) / \partial m_z = 0$, so the field necessary to hold m_z constant is given by

$$\mu_0 H = \frac{\partial E(m_z, 0)}{\partial m_z} \quad (4)$$

We conclude that everything we need to know about the system (the coercivity and energy barrier) is contained in the

function $E(m_z)$, the minimum energy at fixed magnetic moment m_z and zero field. This result is very general. Although we motivated it above by considering domain-wall switching, it describes Stoner-Wohlfarth (S-W) switching as well. This is the limit in which K , A , and M_s are uniform and A is large so $\mathbf{M}(z)$ is uniform. The S-W energy (per unit area, of a grain of length L) is just $E = K L \sin^2 \theta = K L (1 - m_z^2 / m_s^2)$ (here the saturation moment per unit area is $m_s = M_s L$) so the $E(m_z)$ plot is a parabola, as shown in Fig. 1. Note that

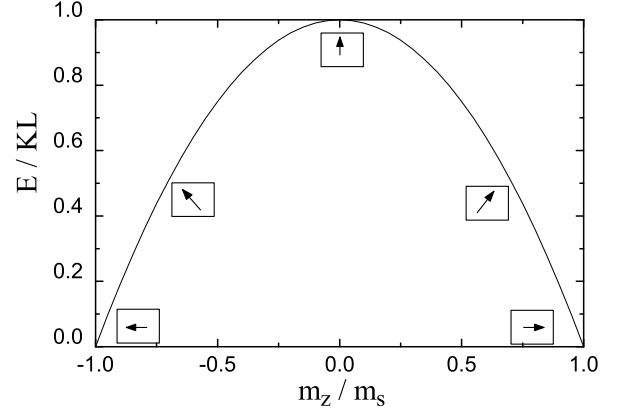


Fig. 1. Energy landscape $E(m_z)$ for a Stoner-Wohlfarth particle.

the slope (the field necessary to switch, Eq. 4) is exactly the Stoner-Wohlfarth switching field $H_{sw} = 2K/M_s$, as expected. [In general, the coercivity is the maximum value of the slope.]

Another virtue of the function $E(m_z)$ is that it has exactly the same interpretation for a particle with a lower exchange constant A as for a S-W (high- A) particle. The behavior depends only on the dimensionless parameter $x = A/KL^2$, which is the square of the ratio of the exchange length to the particle length L . With low x , switching takes place through domain wall motion, and the energy barrier is approximately the domain wall energy. This can be calculated analytically in an infinite system (we will refer to this as the thin-wall approximation, because it is valid when the wall is far from the system boundary and the material properties vary only slightly through the wall) – the thin-wall energy is $4(AK)^{1/2} = 4KLx^{1/2}$. We have developed a numerical minimization program for computing $E(m_z)$ for an arbitrary $K(z)$, and the result for a uniform $K(z)$ is shown in Fig. 2. It can be seen that the energy is indeed constant when m_z is far from its limiting values $\pm m_s \equiv \pm M_s L$, and equal to $4(AK)^{1/2}$. The slope of $E(m_z)$ at the ends is just the domain wall nucleation field. The behavior of the magnetization profile at various times during switching is shown in Fig. 3.

Suess et al[6] have noted that in the thin-wall approximation, the pinning field should remain constant if we choose $K(z) \propto z^2$. In our $E(m_z)$ formulation, this means the slope should be nearly constant. This turns out to be remarkably nearly true numerically, except near the hard end, as shown in Fig. 4.

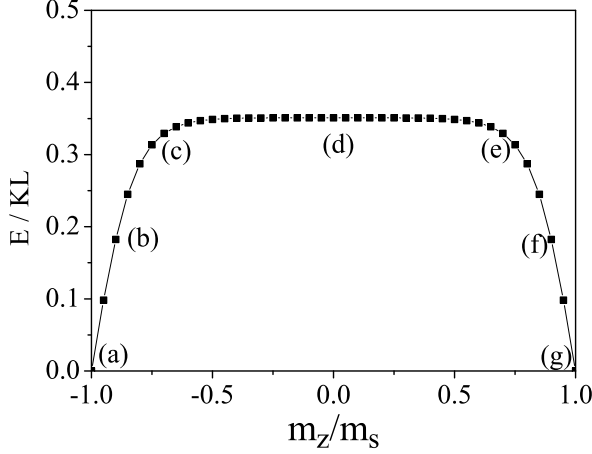


Fig. 2. Energy landscape $E(m_z)$ for a particle with small exchange parameter $x = 0.0076$.

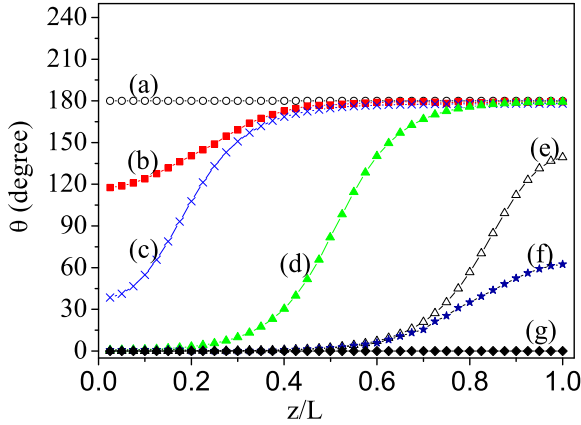


Fig. 3. Magnetization profiles (angle vs. position z) for the particle whose energy landscape is shown in Fig. 2. Labels (a), (b), ... correspond to specific values of m_z shown in Fig. 2.

III. RIGOROUS BOUND ON COERCIVITY FIGURE OF MERIT

Our $E(m_z)$ formulation allows us to prove a completely general (within the assumptions: 1D, quasistatic) result, which is clear geometrically from the $E(m_z)$ graph. If we fix the vertical height (the zero-field barrier E_b) and the horizontal extent ($2m_s$) the minimum possible coercivity (coercivity = maximum slope) is obtained by a straight line, whose slope must be $\mu_0 H = E_b/2m_s$. In terms of the figure of merit, this means $\xi \leq 4$.

Another way of stating this result is that the coercivity of any graded medium cannot be less than 1/4 of the coercivity of a Stoner-Wohlfarth particle (assuming that the latter switches coherently) of the same magnetic moment and energy barrier. [If M_s is constant, fixing the moment is the same as fixing the length L .] Note that the $K(z) \propto z^2$ case (Fig. 4) gives $\xi = 3.23$, close to the theoretical limit, which it approaches as $x \rightarrow 0$.

Note that in this paper we consider only fields along the

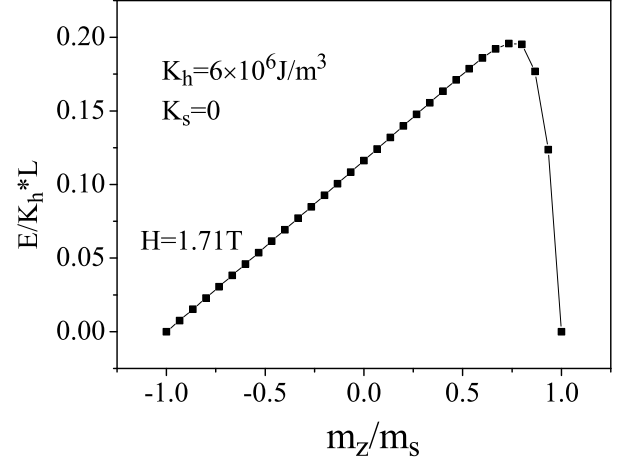


Fig. 4. Energy landscape $E(m_z)$ for the case $K(z) = (K_h/L^2)z^2$, which would be exactly linear in the thin-wall approximation. We have used $K_h = 6 \times 10^6 \text{ J/m}^3$, $L = 22.2 \text{ nm}$, $A = 1.0 \times 10^{-11} \text{ J/m}$, so that the dimensionless exchange parameter $A/K_h L^2 = 0.00338$.

easy axis. Obviously it is worth considering how transverse fields might be useful in switching, since it is known that by using a field at 45° from the axis the Stoner-Wohlfarth switching field is decreased by a factor of 2, so the figure of merit ξ becomes 2. Also, it is likely that the nucleation of a domain wall (which initially requires transverse twisting of the magnetization) can be assisted by a transverse field, so the figure of merit might increase slightly above 4. We should also note that the limit we have established assumes fixed grain length – it may be possible to decrease switching fields beyond the factor of 4 because graded media may make it possible to use longer grains without encountering complicated switching modes such as vortices.

IV. CONCLUSION

We have shown a general bound on the figure of merit of a graded-anisotropy medium, and that this bound (4) is very nearly achieved by $K(z) \propto z^2$. Because the usefulness of such a medium depends on its thermal stability as well as its coercivity, and because of the complex switching mechanism the zero-field switching rate is not completely determined by the energy barrier, an important remaining problem is the more precise calculation of this rate. Although brute force micromagnetic simulation of such slow switching is not practical, work is under way on accelerated sampling techniques for solving this problem[7].

ACKNOWLEDGMENT

This work was supported by NSF MRSEC grant DMR-0213985 and by the DOE Computational Materials Science Network.

REFERENCES

- [1] R. Victora and X. Shen, "Composite media for perpendicular magnetic recording", IEEE Trans. Magn. v. 41, p. 537, 2005.

- [2] J-P. Wang, W. Shen, and J. Bai, "Exchange coupled composite media for perpendicular magnetic recording", IEEE Trans. Magn. v. 41, p. 3181, 2005.
- [3] A. Y. Dobin and H. J. Richter, "Domain wall assisted magnetic recording", App. Phys. Lett. v. 89, p. 062512, 2006.
- [4] D. Suess, "Multilayer exchange spring media for magnetic recording", Appl. Phys. Lett. 89, 113105 (2006).
- [5] A. Aharoni, "Introduction to the theory of Ferromagnetism", Oxford University Press, p. 134, 1996.
- [6] D. Suess, private communication.
- [7] P. B. Visscher and Shuxia Wang, "Rapid simulation of slow switching: a bounce algorithm", paper BE-4, MMM Meeting, January 2007, Baltimore.